Theory and simulation of transverse supermode evolution in a free-electron laser oscillator

Y. Pinhasi, V. Shterngartz, and A. Gover
Department of Physical Electronics, Faculty of Engineering, Tel-Aviv University, Ramat-Aviv, 69978 Tel-Aviv, Israel

(Received 15 February 1996)

We derive the conditions for establishment of a transverse supermode in a free-electron laser (FEL) oscillator, and demonstrate the evolution of a supermode by means of a three-dimensional nonlinear code. Both the analytical formulation and the numerical code are based on coupled-mode theory. The oscillator supermode is a combination of transverse modes that keeps its field profile at any point along the oscillator intact after each round trip, and therefore it is the steady-state result of the oscillation buildup process. In the FEL, as in any laser, the oscillator supermode is identical with the amplifier supermode only if the feedback process is entirely nondispersive. If this is not the case, the steady-state supermode field profile varies along the oscillator axis. The simulations demonstrate that the transverse supermode evolution process is primarily a linear regime process and can be proceeded or even completed before saturation. [S1063-651X(96)13811-3]

PACS number(s): 41.60.Cr

I. INTRODUCTION

Free-electron lasers (FEL) can be employed as coherent sources of electromagnetic radiation in an oscillator configuration [1]. Theoretical studies of radiation buildup in FEL oscillators have been carried out by the Nizhni-Novgorod research group [2–4], the University of California (Santa Barbara) [5,6], and the University of Maryland (UMD) [7–10]. They investigated the nonlinear and saturation processes, taking place in the FEL oscillator. These effects were shown to play an important role in the longitudinal mode-competition process, which leads to the establishment of single-mode lasing. Previous works were carried out in the framework of a one-dimensional (1D) model, assuming a TEM or a single transverse mode of electromagnetic radiation in the resonator.

In optical open resonators and overmoded waveguide cavities, where multitransverse modes may be excited, a three-dimensional (3D) model of FEL interaction is required for adequate description of the oscillation buildup process. Such a model should take into consideration the transverse variations of the electron beam current and the electromagnetic intermode scattering, which originates primarily from the finite transverse dimensions of the gain medium (electron beam). The transverse distribution of the amplified radiation field varies along the interaction region in an FEL amplifier in steady state (guiding). In previous publications [11,12], we showed that there is a combination of transverse modes, which keeps such amplitude and phase relations, so that the field profile of the radiation field (except amplitude and phase) does not change along the interaction region (''an amplifier supermode'').

By contrast, in an oscillator configuration, which we analyze here, the excited radiation, obtained at the output of the FEL interaction region, is fed back to the input. Consequently, the transverse dependence of the circulating radiation field is determined self-consistently by the amplification and feedback processes and evolves gradually into a steady-state distribution (''an oscillator supermode''). A three-dimensional study of the FEL oscillator is required to follow this development.

The 3D analysis of the radiation field excitation used in this paper is based on modal expansion of the total field in terms of transverse eigenmodes of the resonator in which the radiation propagates and a coupled-mode formulation is used. The evolution of the radiation in the resonator and the gain medium (electron current) is studied both in the linear and nonlinear regimes, employing an analytical approach and a 3D simulation code.

II. ANALYSIS OF A MULTITRANSVERSE MODE OSCILLATOR

We first derive analytically a stability criterion for oscillations, assuming that linear gain expressions can be still employed as the oscillator arrives to steady-state operation. This approach is similar to the one employed in general laser theory for estimating the threshold gain required for self-excitation and oscillation startup [13]. It also predicts the frequencies of oscillation (*longitudinal modes*) in stable operation. This analysis is, however, of limited use for predicting the amplitude of oscillations and the power of the output signal.

In laser oscillators, usually many transverse modes can be excited simultaneously and may be coupled to each other. Consequently, one should employ a multimode analysis including feedback conditions in order to formulate the criterion of oscillation.

Assuming a uniform cross-section resonator (usually a waveguide), the total electromagnetic field at every plane z, can be expressed as a sum of a set of transverse (orthogonal) eigenfunctions $\tilde{\boldsymbol{\mathcal{E}}}_q(x,y)$ with related amplitudes $C_q(z)$. At the entrance to the wiggler, the modes are assumed to have initial amplitudes $C_q(0)$ and the total field at $z=0$ is given by

$$\tilde{\mathbf{E}}(x,y,z=0)=\sum_q C_q(0)\tilde{\boldsymbol{\mathcal{E}}}_q(x,y). \tag{1}$$

Passing through the interaction region of the laser, the ''slow varying'' amplitude of each mode is $C_q(z)$, and the total electromagnetic field at the exit of the interaction region can be written as

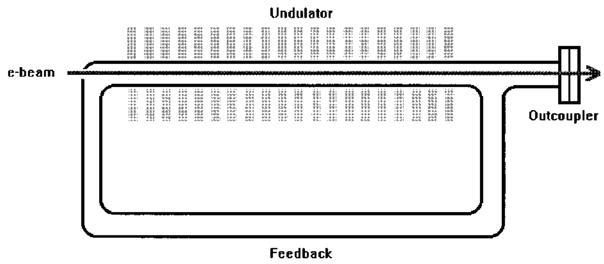

FIG. 1. Schematic illustration of a free-electron laser oscillator.

$$\tilde{\mathbf{E}}(x,y,z=L_w)=\sum_q C_q(L_w)\tilde{\boldsymbol{\mathcal{E}}}_q(x,y)e^{jk_{zq}L_w}, \tag{2}$$

where k_{zq} is the axial propagation constant of transverse mode q. Part of the field is coupled out through the resonator out-coupler and the remainder is reflected and fed back to the input of the FEL amplifier, as shown schematically in Fig. 1.

In the general case, the reflection mirrors can produce intermode scattering and there may be cross coupling between the reflected modes. After a round trip in the resonator the total field circulated back into the entrance of the interaction region is

$$\tilde{\mathbf{E}}(x,y,z=l_c)=\sum_q C_q(l_c)\tilde{\boldsymbol{\mathcal{E}}}_q(x,y)e^{jk_{zq}l_c}. \tag{3}$$

l_c is the total round trip length of the cavity, and $C_q(l_c)$ are the mode amplitudes after a round trip in the resonator and are given by

$$C_q(l_c)=\sum_{q'} \rho_{qq'}C_{q'}(L_w), \tag{4}$$

where $\rho_{qq'}$ are complex reflection coefficients, expressing the intermode scattering (in terms of slow varying amplitudes) of transverse mode q', to mode q, due to the resonator mirrors or any other passive elements in the entire feedback loop. If the scattering matrix from the output of the oscillator back to its input $\underline{\underline{r}}$ is defined [in terms of the fast varying amplitudes $C_q(z)e^{jk_{zq}z}$] by

$$C_q(l_c)e^{jk_{zq}l_c}=\sum_{q'} r_{qq'}C_{q'}(L_w)e^{jk_{zq'}L_w},$$

then comparison with Eq. (4) reveals the relation

$$\rho_{qq'}\equiv r_{qq'}e^{j(k_{zq'}L_w-k_{zq}l_c)}.$$

The expression for the total circulated field is found from Eqs. (3) and (4):

$$\tilde{\mathbf{E}}(x,y,z=l_c)=\sum_q \left[\sum_{q'} \rho_{qq'}C_{q'}(L_w)\right]\tilde{\boldsymbol{\mathcal{E}}}_q(x,y)e^{jk_{zq}l_c}. \tag{5}$$

When the oscillator arrives to its steady-state regime of operation, the initial field at $z=0$ must be equal to the circulated field after a round-trip $z=l_c$, i.e.,

$$\mathbf{E}(x,y,z=0)=\mathbf{E}(x,y,z=l_c). \tag{6}$$

By substituting in Eq. (6) the expressions (1) and (5) for the fields, and scalar multiplying both sides of the equation by the eigenfunction $\tilde{\boldsymbol{\mathcal{E}}}_q(x,y)$, one obtains the steady-state oscillation condition:

$$C_q(0)=e^{jk_{zq}l_c}\sum_{q'} \rho_{qq'}C_{q'}(L_w). \tag{7}$$

It was shown in the coupled-mode analysis of the FEL amplifier carried out in [11,12] that the amplitude of the transverse modes at the output of the FEL interaction region can be written in terms of the gain matrix $\underline{\underline{\Gamma}}(L_w)$ of the FEL,

$$\underline{C}(L_w)=\underline{\underline{\Gamma}}(L_w)\underline{C}(0). \tag{8}$$

Substituting Eq. (8) in Eq. (7), we derive a set of equations for the amplitudes of the modes in steady-state operation:

$$C_q(0)=e^{jk_{zq}l_c}\sum_{q'} \rho_{qq'}\sum_{q''} \Gamma_{q'q''}(L_w)C_{q''}(0). \tag{9}$$

The last set of equations (9) can be written in a compact matrix form:

$$[e^{j\underline{\underline{K}}_z l_c}\underline{\underline{\rho}}\,\underline{\underline{\Gamma}}(L_w)-\underline{\underline{I}}]\underline{C}(0)=0, \tag{10}$$

where the matrix $\underline{\underline{K}}_z$ is a diagonal matrix with the wave numbers k_{zq} on its diagonal. The condition for a nontrivial solution for $\underline{C}(0)$ is vanishing of the determinant:

$$|e^{j\underline{\underline{K}}_z l_c}\underline{\underline{\rho}}\,\underline{\underline{\Gamma}}(L_w)-\underline{\underline{I}}|=0. \tag{11}$$

This is a generalized oscillation criterion for the case where a number of transverse modes are excited in the resonator. Note that this criterion can be generalized also for lasers with two mirror resonators of reflectivity matrices $\underline{\underline{\rho}}_1$ and $\underline{\underline{\rho}}_2$, for which Eq. (11) can be written as $|e^{j\underline{\underline{K}}_z l_c}\underline{\underline{\rho}}_1\underline{\underline{\Gamma}}_1(L_w)\underline{\underline{\rho}}_2\underline{\underline{\Gamma}}_2(L_w)-\underline{\underline{I}}|=0$. It is an extension to the criterion derived for single transverse mode laser oscillators (where the gain, wave number, and reflection coefficient are scalars) [13], which requires the round-trip gain and oscillation frequency to satisfy $\rho_q\Gamma_q(L_w)e^{jk_{zq}l_c}=1$. Using analytical expressions for $\underline{\underline{\Gamma}}(L_w)$, derived in [12], the 3D criterion for oscillation (11) is useful for estimating more accurately the FEL oscillator threshold conditions and oscillation frequencies.

Evidently, the expansion modes (the free-space or waveguide modes), which are the eigenmodes of the "cold" resonator without a gain medium and feedback, are in general not the eigenmodes of the FEL oscillator. These modes are coupled to each other by the gain medium (e beam) of the FEL amplifier and may also be interscattered by the reflection mirror.

If a similarity transformation that diagonalizes the round-trip matrix $e^{j\underline{\underline{K}}_z l_c}\underline{\underline{\rho}}\,\underline{\underline{\Gamma}}(L_w)$ exists, the oscillator will reach a stable regime of operation. In the following we show that finding such a similarity transformation is equivalent to solving for a new set of independent modes that become the steady-state eigenmodes (*supermodes*) of the oscillator.

III. THE "SUPERMODES" OF THE FEL OSCILLATOR

To derive the field profile of the oscillator supermodes, we employ a linear transformation which transforms the coupled system of modes to an uncoupled one [14,15]. In every cross section, each of the transverse (free-space or waveguide) modes can be written as a linear combination of a new set of uncoupled eigenmodes with amplitudes $\underline{U}(z)$. The relation between the two representations at $z=0$ is given through the linear transformation:

$$\underline{C}(0)=\underline{\underline{T}}\,\underline{U}(0). \tag{12}$$

This transformation is used together with Eq. (10) to derive the steady-state condition for the supermodes:

$$\underline{U}(0)=\underline{\underline{T}}^{-1}[e^{j\underline{\underline{K}}_z l_c}\underline{\underline{\rho}}\,\underline{\underline{\Gamma}}(L_w)]\underline{\underline{T}}\,\underline{U}(0). \tag{13}$$

The above equation (13) is satisfied when the similarity transformation $\underline{\underline{T}}^{-1}[e^{j\underline{\underline{K}}_z l_c}\underline{\underline{\rho}}\,\underline{\underline{\Gamma}}(L_w)]\underline{\underline{T}}$ produces a diagonal unit matrix. In that case the linear transformation $\underline{\underline{T}}$ represents the superposition of cavity modes [of amplitudes $C_q(0)$] at $z=0$ that keeps its transverse features every round trip in the resonator. Note that this superposition may vary along the resonator.

Unlike the amplifier case, which is characterized by an axial translational symmetry where the supermodes maintaining their transverse field profile and polarization along the interaction region, the only symmetry which exists at steady-state operation of the oscillator is a round-trip periodicity of the circulating field, which is analogous to periodic translational symmetry with periodicity l_c. Thus, the supermodes of the oscillator do not, in general, keep their transverse profile unchanged along the resonator.

IV. THE OSCILLATOR SUPERMODES—DEGENERATE CASE

In order to see better the relations between the oscillator and amplifier supermodes, it helps to substitute $\underline{\underline{\rho}}$ in Eq. (13) in terms of the scattering matrix $\underline{\underline{r}}$:

$$\underline{U}(0)=\underline{\underline{T}}^{-1}[\underline{\underline{r}}e^{j\underline{\underline{K}}_z L_w}\underline{\underline{\Gamma}}(L_w)]\underline{\underline{T}}\,\underline{U}(0). \tag{14}$$

In the special case when $\underline{\underline{r}}=r\underline{\underline{I}}$ is a scalar, namely the feedback is nondispersive and all the modes are reflected indiscriminately (same phase and amplitude coefficient) and without interscattering, Eq. (14) can then be written as

$$\underline{U}(0)=r\underline{\underline{T}}^{-1}[e^{j\underline{\underline{K}}_z L_w}\underline{\underline{\Gamma}}(L_w)]\underline{\underline{T}}\,\underline{U}(0). \tag{15}$$

The transformation matrix $\underline{\underline{T}}$ is exactly the transformation required in order to derive the supermodes of the free-electron laser amplifier [11,12]. It is identified to be the similarity matrix that diagonalizes the coupled mode gain matrix $e^{j\underline{\underline{K}}_z L_w}\underline{\underline{\Gamma}}(L_w)$ of the amplifier section of the FEL. The resulting diagonal matrix consists of the gain coefficients $\Lambda_i(L_w)=U_i(L_w)/U_i(0)$ of the uncoupled normal modes of the FEL. They are the eigenvalues of the coupled-mode gain matrix and are found from the algebraic equation:

$$|e^{j\underline{\underline{K}}_z L_w}\underline{\underline{\Gamma}}(L_w)-\Lambda(L_w)\underline{\underline{I}}|=0. \tag{16}$$

An example for such a special case is when the transverse modes excited in the oscillator are degenerate in their longitudinal wave number k_{zq} and are reflected by the resonator mirrors indiscriminately (without interscattering) and with equal reflection coefficient ρ. Equation (13) can then be written as

$$\underline{U}(0)=\rho e^{jk_z l_c}[\underline{\underline{T}}^{-1}\underline{\underline{\Gamma}}(L_w)\underline{\underline{T}}]\underline{U}(0). \tag{17}$$

The transformation matrix $\underline{\underline{T}}$ used in Eq. (13) to find the supermodes of the oscillator is identified to be the similarity matrix that diagonalizes the gain matrix $\underline{\underline{\Gamma}}(L_w)$ of the amplifier section of the FEL. The eigenvalues $\Lambda_i(L_w)$ of the coupled-mode gain matrix in that case are found from the algebraic equation:

$$|\underline{\underline{\Gamma}}(L_w)-\Lambda(L_w)\underline{\underline{I}}|=0. \tag{18}$$

Given the eigenvalues $\Lambda_i(L_w)$, the stability criterion for each of the uncoupled oscillator supermodes can be written as $\rho\Lambda_i(L_w)e^{jk_z l_c}=1$, similar to that derived in a single (transverse) mode analysis of oscillators. Employing an explicit expression for the gain matrix $\Lambda_i(L_w)$ which was derived analytically in [12] for the supermode in the linear regime, the solution of the stability criterion (18) determines the lasing gain threshold and the operating frequencies.

V. THREE-DIMENSIONAL SIMULATION OF FEL OSCILLATOR

In order to demonstrate the evolution of the electromagnetic radiation field in a multitransverse mode free-electron laser oscillator into a supermode, we employ a three-dimensional computer program FEM3D simulating the FEL amplifier operation in the linear and nonlinear regimes and an appropriate algorithm for feedback process.

The FEL amplification code is based on a modal expansion of the total electromagnetic field in terms of transverse waveguide modes [Eq. (2)], and solves a self-consistent system of electron force equations and electromagnetic excitation equations [16].

A set of excitation equations describes the evolution of amplitude $C_q(z)$ of each transverse mode along the interaction region:

$$\frac{d}{dz}\,C_q(z)=\frac{1}{2\mathcal{P}_q}\,e^{-jk_{zq}z}\,\frac{I_0}{N}\sum_{i=1}^{N}\,\frac{1}{v_{zi}}\,\mathbf{v}_i\cdot\tilde{\mathcal{E}}_q^*(x_i,y_i)e^{j\omega_s t_i(z)}, \tag{19}$$

where $\mathcal{P}_q=\frac{1}{2}\mathrm{Re}\int\int[\tilde{\mathcal{E}}_{\perp q}\times\tilde{\boldsymbol{\mathcal{H}}}_{\perp q}^*]\cdot\hat{\mathbf{z}}dx\,dy$ is the power normalization of the propagating mode and I_0 is the electron beam current. (x_i,y_i) are the transverse coordinates of particle i, and $\mathbf{v}_i$ is its velocity vector. The dynamics of each of the N particles in the simulation is described by the force equation

$$\frac{d}{dz}\,(\gamma_i\mathbf{v}_i)=-\frac{e}{m}\,\frac{1}{v_{zi}}\,[\mathbf{E}+\mathbf{v}_i\times\mathbf{B}], \tag{20}$$

where the relativistic factor γ_i is found from

$$\frac{d\gamma_i}{dz}=-\frac{e}{mc^2}\,\frac{1}{v_{zi}}\,\mathbf{v}_i\cdot\mathbf{E}. \tag{21}$$

The time it takes a particle to arrive at a position z is

$$t_i(z)=t_{0_i}+\int_0^z \frac{1}{v_{zi}(z')}\,dz', \tag{22}$$

where t_{0i} is the time when the ith particle entered at $z=0$.

The entrance time t_{0i} of the N particles is determined to be distributed uniformly over the time period $T=2\pi/\omega_s$ of the signal, i.e., $t_{0i}=(2\pi/\omega_s)(i-1)/N$ for $i=1,2,\ldots,N$. The assumption is that the problem can be described correctly by a steady-state statistical distribution of the electron initial time t_{0i}, that is, periodic with period T. Namely, it is enough to sample electrons in one optical period $T=2\pi/\omega_s$, and the field solutions in any other period will be the same. This assumption is good only for stimulated and prebunching radiation, not spontaneous emission and amplified spontaneous emission problems, where the sampling time of electrons must be a slippage time $\tau_{sp}=L_\omega/v_{z0}-L_w/v_g$ (v_{z0} is the axial velocity of the electrons and v_g is the group velocity of the electromagnetic radiation in the cavity). At the present version of the simulation code, the N particles are injected into the interaction region homogeneously, via equal time intervals. The code can be modified to simulate radiation buildup process in a prebunched FEL oscillator by inserting the particles into the interaction region with a nonuniform distribution over the time period T. We found that reliable results are obtained when $N>20$ particles are taken in the simulation along the longitudinal dimension.

The oscillation buildup process is followed round trip after round trip until steady state is achieved, assuming approximate knowledge of the oscillation frequency at steady state *a priori*. Such an assumption is reasonable in cases where the oscillator is expected to arrive to a single frequency stable operation at steady state. In cases where the number of longitudinal modes under the gain curve is small (short resonator or use of frequency filtering structure), the model will describe the real oscillation buildup process. In other cases, it will describe correctly only the supermode profile that is attained at steady state.

VI. TRANSVERSE MODE EVOLUTION IN THE FEL OSCILLATOR

We first show the calculation of the supermode in a specific example based on the analytical theory in the linear regime. The example presented here is of the electrostatic accelerator free-electron maser (FEM) now being developed in Israel [17,18]. The basic parameters of the FEM are given in Table I. The waveguide is a $1.5\times1.5\ \mathrm{cm}^2$ rectangular waveguide in which the fundamental TE_{01} mode and the degenerate TE_{21} and TM_{21} modes are found to be within the frequency range of operation. Other modes are too far from phase matching and do not contribute to the interaction.

Figure 2 illustrates the small-signal gain curves of the TE_{01}, TE_{21}, and TM_{21} modes, excited in the FEL amplifier operating in the linear regime. The curves are calculated running the numerical simulation in the small-signal regime.

The results of single mode gain calculations (disabling coupling between the modes in the gain calculations) are given as dashed lines. Since the TE_{21} and TM_{21} modes have the same wave number, they operate in the same frequency range, and can strongly couple to each other. The gain curve of their resultant supermode (in the amplifier sense) is shown as a continuous line. This mode, found from coupled-mode theory, was identified in this particular case as the linearly polarized LP_{21} mode of the rectangular waveguide [19], which is purely polarized in the wiggling dimension. Inspection of the gain curve reveals that the highest gain is obtained at a frequency $f=116$ GHz, where the linearly polarized mode LP_{21} exhibits maximum gain. The UMD theory [6–10] predicts that at steady state, the oscillator will arrive to single frequency operation, and the longitudinal mode, which will ''survive'' the mode competition process, is expected to oscillate near this maximum gain frequency. Hence we chose this frequency to run the example in the present multitransverse modes, single-longitudinal mode simulation.

TABLE I. Parameters of the tandem electrostatic accelerator FEL.

Accelerator:	
Electron beam energy	$E_k=2$ MeV
Beam current	$I_0=1$ A
Wiggler:	
Magnetostatic planar wiggler	
Magnetic induction	$B_w=3$ K G
Period length	$\lambda_w=4.4$ cm
Number of periods	$N_w=20$
Waveguide:	
Rectangular waveguide	$1.5\times1.5\ \mathrm{cm}^2$

We now report also a complete nonlinear numerical simulation of the process of radiation buildup in the FEL oscillator. Starting from a low level of initial power, the radiation obtained at the output of the FEL amplifier at each stage is fed back to its input, as described by Eq. (7), assuming that there is no cross coupling between the modes due to the mirrors of the resonator. The phase shift for the degenerate TE_{21} and TM_{21} is assumed to be $2m\pi$ (the phase shift of the TE_{01} is determined by its wave number and the length of the feedback loop). Neglecting at this time multilongitudinal mode competition, we assume operation at a single frequency corresponding to the maximum linear gain of the TE_{21} and the TM_{21} modes and uniform power reflectivity of $\mathcal{R}=|\rho|^2=90\%$ for each of the transverse modes. Internal

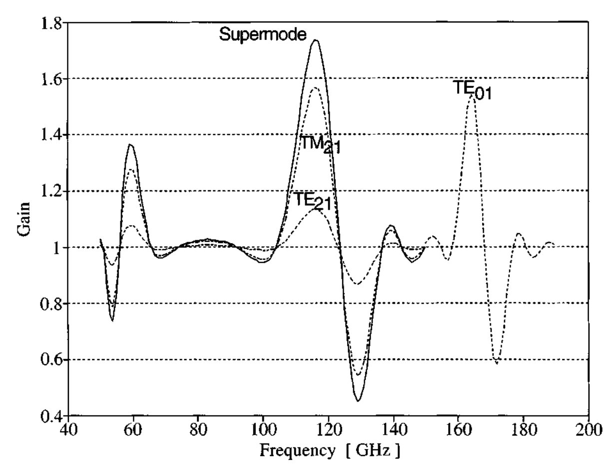

FIG. 2. Small-signal gain curves of the FEL amplifier.

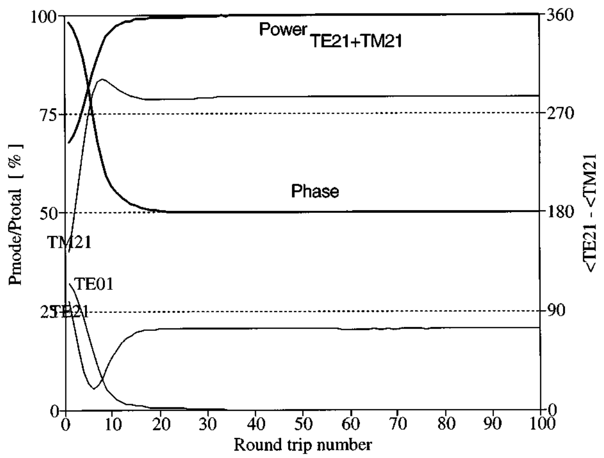

FIG. 3. Relative circulating power and phase evolution of transverse waveguide modes starting from equal power and the same phase in a FEL oscillator.

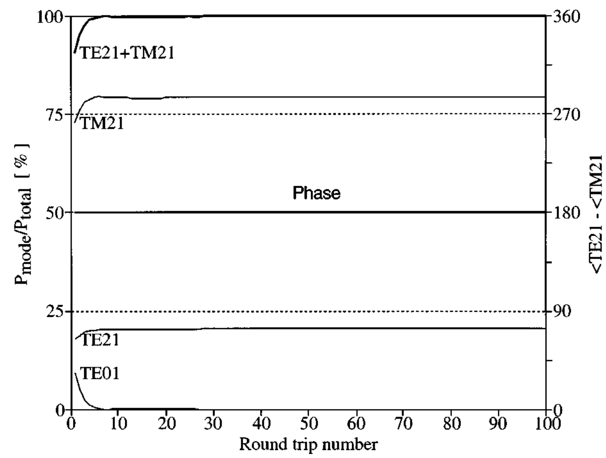

FIG. 5. Relative circulating power and phase evolution of transverse waveguide modes starting from power and phase relations that exhibit the FEL oscillator supermode.

waveguide losses are neglected.

At the first round trip, the fundamental TE_{01} mode and the degenerate TE_{21} and TM_{21} modes were assigned equal initial power and phase. The initial power was determined to be sufficiently small to avoid nonlinear effects on the first traversals. Graphs of the power carried by each of the individual modes relative to the total power circulating in the oscillator,

$$\frac{P_{\text{mode}}}{P_{\text{total}}}=\frac{|C_q(L_w)|^2\mathcal{P}_q}{\Sigma_{q'}|C_{q'}(L_w)|^2\mathcal{P}_{q'}},$$

is shown in Fig. 3. The phase relation between the degenerate TE_{21} and TM_{21} modes is also drawn. The evolution of the single-pass gain of the individual modes

$$G_q=\frac{|C_q(L_w)|^2}{|C_q(0)|^2}$$

and the total gain

$$G_{\text{total}}=\frac{P(L_w)}{P(0)}=\frac{\Sigma_q|C_q(L_w)|^2\mathcal{P}_q}{\Sigma_q|C_q(0)|^2\mathcal{P}_q}$$

as a function of round-trip number is shown in Fig. 4.

During several round trips, the radiation power is still small and the FEL is operating in the linear regime. The gain of the coupled TE_{21} and TM_{21} modes is self-adjusted until the power is shared in a combination that corresponds to the LP_{21} supermode. (Note that in this process the gain of the TE_{21} is initially less than 1 and then excessively high, until it coincided with the gain of the TM_{21} and the supermode.) The nonsynchronous fundamental TE_{01} mode does not contribute much to the interaction, but it experiences a substantially high gain during the oscillation buildup period. As the circulated power grows, the oscillator enters the nonlinear regime, and the gain decreases. In this regime, the amplitude growth of the modes restrains until saturation is reached. Saturation is characterized by a constant FEL gain, equal to the transmission losses of the cavity (in the present simulation the gain $G=1/\mathcal{R}=1.1$). Observe that the phase difference changes until the TE_{21} and TM_{21} modes lock in antiphase. This demonstrates the transverse mode evolution towards

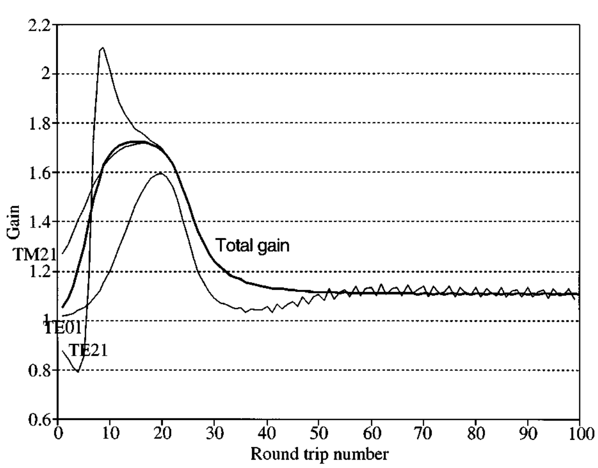

FIG. 4. Gain evolution of transverse waveguide modes starting from equal power and the same phase in a FEL oscillator.

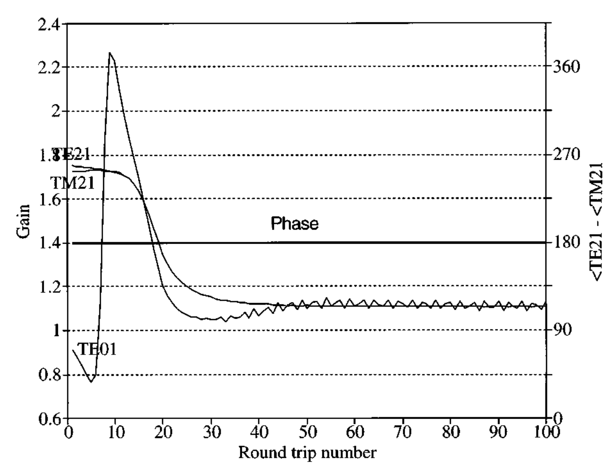

FIG. 6. Gain evolution of transverse modes starting from power and phase relations that exhibit the FEL oscillator supermode.

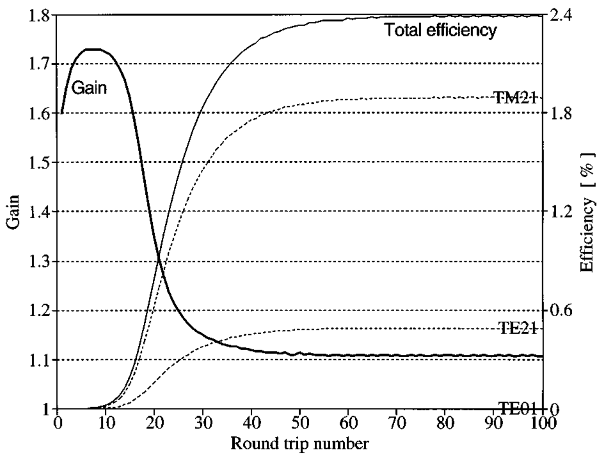

FIG. 7. Total gain and efficiency of the FEL oscillator.

generation of the LP_{21} supermode, which is an antiphase combination of the TE_{21} and TM_{21} modes. This supermode is the steady-state eigenmode of the FEL oscillator.

It is important to note that the process of the supermode buildup starts well before the onset of saturation. Contrary to the longitudinal mode competition process, which is an entirely nonlinear (saturation regime) effect [2–10], the transverse mode interaction process takes place also in the linear regime.

One can also start the power circulation initially with the combination that produces the supermode right from the beginning. The linearly polarized LP_{mn} supermode is obtained in a rectangular $a\times b$ waveguide, when the degenerate TE_{mn} and TM_{mn} modes are excited simultaneously with a power relation $P_{TM}/P_{TE}=(k^2/k_z^2)(k_x^2/k_y^3)$, where $k_x=m\pi/a$ and $k_y=n\pi/b$, and with a phase difference of 180° [19]. In that case the degenerate TE_{21} and TM_{21} modes will keep their relative power and phase relations starting from the first round-trip traversals of oscillation buildup, as demonstrated in Fig. 5, and will enter together into the saturation regime. Since the power ratio between the degenerate modes is conserved during the radiation buildup, the individual TE_{21} and TM_{21} modes have equal gain also in the linear regime (see Fig. 6). The gain of the supermode is equal to the total gain of the FEL.

The simulation confirms numerically the prediction of the analytical model, namely, that when the feedback is nondispersive, the *oscillator supermode* at steady state is identical to the *amplifier supermode*, and produces the same supermode solution (the LP_{21} mode) that was predicted with the analytical model.

In Fig. 7 the output efficiency of individual modes and the total gain are shown as a function of the number of round trips for the case where the supermode is set initially. The gain variation during the first few round trips can be associated with the interference of the TE_{01} mode, which has initially the same power as the TE_{21} mode. When the TE_{01} mode diminishes, the total gain returns to the small-signal gain of the supermode (Fig. 6) and then falls down in the nonlinear regime to the same saturation value $G=1/\mathcal{R}=1.1$.

VII. SUMMARY AND CONCLUSIONS

Transverse mode evolution in a FEL oscillator is discussed. A generalized multimode oscillation condition is derived, and the eigenmodes of the oscillator in steady state are found. These modes are identified to be the transverse ''supermodes'' of the FEL amplifier if the feedback system is nondispersive and does not produce intermode scattering.

The evolution of the modes and the formation of a ''supermode'' is examined using a nonlinear multitransverse modes code. The formation of the ''supermode'' may take place in the linear or in the nonlinear regime, depending on the internal losses and the outcoupling transmission.

[1] V. L. Granatstein and I. Alexoff, *High Power Microwave Sources* (Artech House, Boston, 1987).
[2] V. L. Bratman, N. S. Ginzburg, and M. I. Petelin, Zh. Eksp. Teor. Fiz. **76**, 936 (1979) [Sov. Phys. JETP **49**, 469 (1979)].
[3] V. L. Bratman, N. S. Ginzburg, and M. I. Petelin, Opt. Commun. **30**, 409 (1979).
[4] I. Kimmel and L. Elias, Phys. Rev. A **35**, 3818 (1987).
[5] I. Kimmel and L. Elias, Phys. Rev. A **38**, 2889 (1988).
[6] B. Levush and T. M. Antonsen, Nucl. Instrum. Methods Phys. Res. A **272**, 375 (1988).
[7] B. Levush and T. M. Antonsen, Nucl. Instrum. Methods Phys. Res. A **285**, 136 (1988).
[8] T. M. Antonsen and B. Levush, Phys. Fluids B **1**, 1097 (1989).
[9] T. M. Antonsen and B. Levush, Phys. Rev. Lett. **62**, 1488 (1989).
[10] B. Levush and T. M. Antonsen, IEEE Trans. Plasma Sci. **18**, 260 (1990).
[11] Y. Pinhasi *et al.*, Nucl. Instrum. Methods Phys. Res. A **318**, 523 (1992).
[12] Y. Pinhasi and A. Gover, Phys. Rev. E **51**, 2472 (1995).
[13] A. Yariv, *Optical Electronics*, 4th ed. (Holt, Rinehart & Winston, New York, 1991).
[14] W. H. Louisell, *Coupled Mode and Parametric Electronics* (Wiley, New York, 1960).
[15] J. H. Wilkinson, *The Algebraic Eigenvalue Problem* (Calderon, Oxford, 1965).
[16] Y. Pinhasi, M. Cohen, and A. Gover, Int. J. Electron. **78**, 581 (1995).
[17] M. Cohen, M. Draznin, A. Goldring, A. Gover, Y. Pinhasi, J. Wachtell, Y. Yakover, J. Sokolowski, B. Mandelnbaum, Y. Shiloh, G. Hazak, L. M. Levine, and O. Shahal, Nucl. Instrum. Methods Phys. Res. A **341**, ABS57 (1994).
[18] A. Arensburg, M. Cohen, M. Draznin, A. Eichenbaum, A. Gover, Y. Pinhasi, V. Shterngartz, I. M. Yakover, J. Sokolovski, B. Mandelbaum, A. Rosenberg, J. Shiloh, G. Hasak, L. A. Levine, O. Shahal, M. Shapira, and V. Granatstein, Nucl. Instrum. Methods Phys. Res. A **358**, ABS7 (1995).
[19] L. Lewin, *Theory of Waveguides* (Butterworths, London, 1975).